SPECIAL RELATIVITY 001
2/1/2023
The following is a simplified derivation of the relativity of space and time that is not rigorous but has some heuristic value.

(a) The magnetic force law on a charge +*q* moving in the +*x*-direction with velocity v_x, in the presence of an instantaneous magnetic field in the +*z*-direction, B_z, as seen by an observer stationary with respect to B_z;

(b) Faraday's Law to establish an electric field in the -y-direction as seen by an observer moving with the charge;

(c) A rationalization of the invariance of the speed of light *c*;

(d) A Pythagoras construction to derive the relativity of time and distance.

Relativity of E and B

(1) The magnetic force on a charge +*q* moving in the +*x*-direction with velocity v_x in the presence of a magnetic field B_z in the +*z*-direction, is in the −*y* direction: $F_y = -qv_xB_z = -q\left(x/t\right)B_z$. The negative sign arises from application of the right hand rule.

(2) For an observer moving with the charge, *x* is constant, and the magnetic field is seen to change with time: $F_y = -qx\left(\Delta B_z / \Delta t\right)$. Faraday's Law is

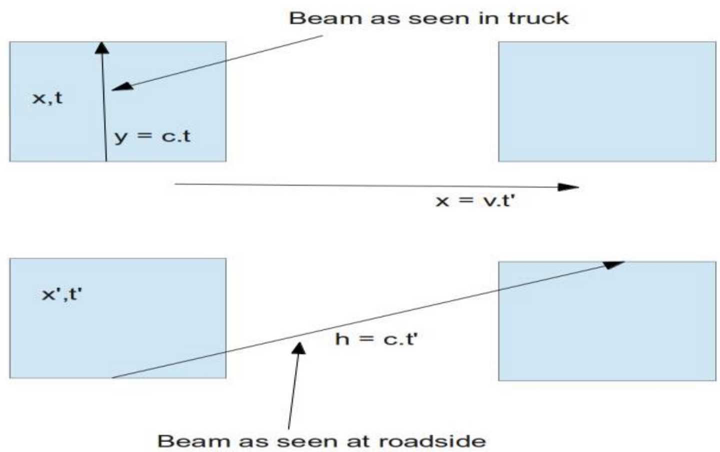
$\varepsilon = -A_{xy}\left(\Delta B_z / \Delta t\right) \Rightarrow \left(\Delta B_z / \Delta t\right) = -\varepsilon / A_{xy} = -\varepsilon / \Delta x \Delta y$ so that the force law becomes

$F_y = -q\Delta x\left(-\varepsilon / \Delta x\Delta y\right) = q\left(\varepsilon / \Delta y\right) = -qE_y$.

Thus both observers see a force in the –y direction but they interpret it differently - the stationary observer sees a constant magnetic field in the +*z* direction but the moving observer sees a constant electric field in the −*y* direction. Thus *E* and *B* are relative.

Constancy of c

Visible light, and other forms of radiation such as radio waves, microwaves, x-rays etc., are sine (or cosine) waves of *E* and *B* that are perpendicular to each other and to the propagation direction of the radiation. For an observer moving with constant velocity relative to another observer (at rest on the ground say) some of what the stationary observer sees as a magnetic field is seen by the moving observer as an electric field, and some of what the moving observer sees as an electric field is seen by the moving observer as a changing magnetic field (see *Relativity of E and B* above). The rigorous result is that each observer sees an identical light wave with the same oscillating *E* and *B* fields. Since the speed of light is given by *E/B* each observer sees the same speed of propagation. Thus the speed of light *c* is seen to be exactly the same by observers that are moving relative to each other with a constant relative velocity. If they are moving at a non-constant relative velocity, i.e. at a non-zero relative acceleration, all hell breaks loose and things get extremely complicated and the General Theory of Relativity must be considered.



Relativity of Time and Distance
Relativity of Time

Consider a truck moving at speed v in the $+x$ direction (see figure above) as seen by two observers, one on the roadside and the other inside the truck. To keep track of things a prime is added to the distance and time measured by the roadside observer. Thus distance and time measured by the observer in the truck are denoted by$\{x,y,z,t\}$, and those measured by the roadside observer by $\{x',y',z',t'\}$. The person in the truck shines a light beam vertically upward (relative to the truck) from the floor of the truck to the ceiling and measures the time t it takes to travel the height h as (s)he measures it. The roadside observer sees the light beam travel at an angle to the vertical because in the time taken for the light to reach the ceiling the truck has moved in the horizontal x' direction. These two viewpoints are also illustrated in the figure above.

Application of Pythagoras to the right triangle whose sides are the height of the truck *h*, the distance x' the truck has traveled, and the hypotenuse *h′* of the light beam as seen by the roadside observer yields

$$h^2 = x^2 + y^2 \Rightarrow c^2t'^2 = v^2t'^2 + c^2t^2 \Rightarrow t'^2\left(c^2 - v^2\right) = c^2t^2 \Rightarrow t'^2 = t^2 / \left(1 - v^2/c^2\right)$$

$$\Rightarrow t' = \frac{t}{\left(1 - v^2/c^2\right)^{1/2}}. \qquad (1)$$

Now consider the physics of eq. (1).

(1) The time interval *t′* measured by the roadside observer is greater than the time interval *t* measured by the observer in the truck. This prediction has been confirmed experimentally. The difference between *t′* and *t* is insignificant when *v/c* is very small, which is what we experience in everyday life because the speed of light *c* is about 2/3 of a billion miles per hour. Thus at 100 mph the difference between *t* and *t′* is about 1 part in 100 trillion. The only unique time interval is that measured by an observer moving with an object since all other time intervals depend on the speed of the object relative to the observer. The time *t* is referred to as the proper time - it is often denoted by *τ*.

Relativity of Distance

Both observers agree about the values of *c* and $|v|$. Thus $x/t = x'/t'$ and the relation between *t* and *t′* then implies $x' = x / \left(1 - v^2/c^2\right)^{1/2}$. In this case the "proper" distance is $x' \geq x$. Lengths perpendicular to *v* are not affected because the perpendicular components of *v* are zero.