SPECIAL RELATIVITY 001

We offer a simplified derivation of the relativity of space and time that uses:

(1) The magnetic force law on a charge $+q$ moving in the $+x$-direction with velocity v_x, in the presence of a magnetic field in the $+z$-direction, B_z, as seen by an observer stationary with respect to B_z;

(2) Faraday's Law to establish an electric field in the -y-direction as seen by an observer moving with the charge;

(3) A rationalization of the invariance of the speed of light c;

(4) A Pythagoras construction to derive the relativity of time and distance.

Relativity of E and B

(1) The magnetic force on a charge $+q$ moving in the $+x$-direction with velocity v_x in the presence of a magnetic field B_z in the $+z$-direction, is in the $-y$ direction: $F_y = -qv_x B_z = -q\left(x/t\right)B_z$. The negative sign arises from application of the right hand rule.

(2) For an observer moving with the charge, x is constant and the magnetic field is seen to change with time: $F_y = -qx\left(\Delta B_z / \Delta t\right)$. Faraday's Law is

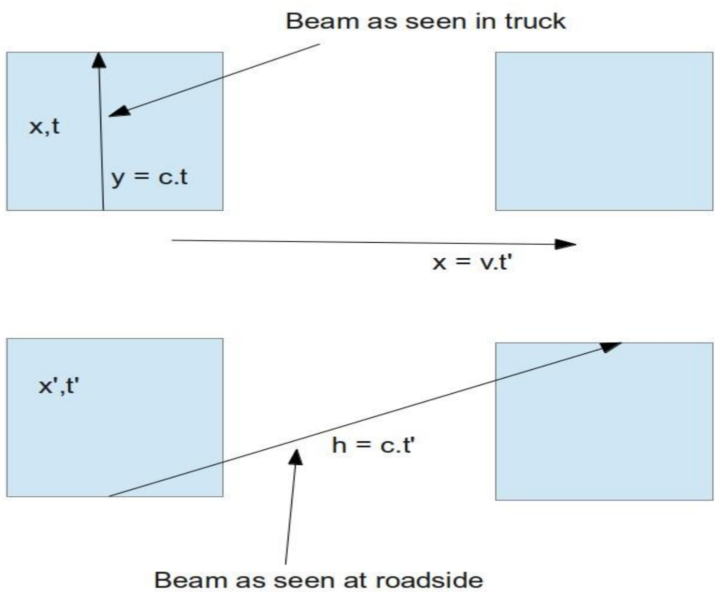
$\varepsilon = -A_{xy}\left(\Delta B_z / \Delta t\right) \Rightarrow \left(\Delta B_z / \Delta t\right) = -\varepsilon / A_{xy} = -\varepsilon / \Delta x \Delta y$ so that the force law becomes

$F_y = -q\Delta x\left(-\varepsilon / \Delta x \Delta y\right) = q\left(\varepsilon / \Delta y\right) = -qE_y$.

Thus both observers see a force in the –y direction but they interpret it differently - the stationary observer sees a constant magnetic field in the $+z$ direction but the moving observer sees a constant electric field in the $-y$ direction. Thus E and B are relative.

Constancy of c

Visible light, and other forms of radiation such as radio waves, microwaves, x-rays etc., are sine (or cosine) waves of E and B that are perpendicular to each other and to the propagation direction of the radiation. For an observer moving with constant velocity relative to another observer (at rest on the ground say) some of what the stationary observer sees as a changing electric field is seen by the moving observer as a changing magnetic field, and some of what the stationary observer sees as a changing magnetic field is seen by the moving observer as a changing electric field. Thus each observer sees an identical light wave with the same oscillating E and B fields. Since the speed of light is given by E/B each observer sees the same speed of propagation. Thus the speed of light c is seen to be exactly the same by observers that are moving relative to each other with a constant relative velocity. If they are moving at a non-constant relative velocity, i.e. at a non-zero relative acceleration, things get extremely complicated and we must consider the General Theory of Relativity.



Relativity of Time and Distance

Relativity of Time

Consider a truck moving at speed v in the $+x$ direction (see figure). We consider two observers, one on the roadside and the other inside the truck. To keep track of things we add a prime to the distance and time measured by the roadside observer. Distances and time measured by the observer in the truck are denoted by$\{x,y,z,t\}$, and those measured by the roadside observer by $\{x',y',z',t'\}$. The person in the truck shines a light beam vertically upward (relative to the truck) from the floor of the truck to the ceiling and measures the time t it takes to travel the height y as (s)he measures it. The roadside observer sees the light beam travel at an angle to the vertical because in the time taken for the light to reach the ceiling the truck has moved in the horizontal x direction. These two viewpoints are illustrated in the figure.

Now apply Pythagoras to the right triangle whose sides are the height of the truck and the distance the truck has travelled, and whose hypotenuse is the non-vertical path of the light beam as seen by the roadside observer. These three distances are:

(i) Horizontal distance x' moved by the truck as measured by the roadside observer: $x' = vt'$.

(ii) Vertical height y of truck as measured by the observer in the truck: $y = ct$.

(iii) Distance h' travelled by the light beam measured by the roadside observer: $h' = ct'$.

Pythagoras then yields:

$$h^2 = x^2 + y^2 \Rightarrow c^2t'^2 = v^2t'^2 + c^2t^2 \Rightarrow t'^2\left(c^2 - v^2\right) = c^2t^2 \Rightarrow t'^2 = t^2 / \left(1 - v^2/c^2\right)$$

$$\Rightarrow t' = \frac{t}{\left(1 - v^2/c^2\right)^{1/2}}.$$

Now consider the physics of the last equation.

(1) The time interval t' measured by the roadside observer is greater than the time interval t measured by the observer in the truck. This prediction has been confirmed experimentally. The difference between t' and t is insignificant when v/c is very small, which is what we experience in everyday life because the speed of light c is about 2/3 of a billion miles per hour. Thus at 100 mph the difference between t and t' is about 1 part in 100 trillion - no wonder the difference between t and t' is not noticed.

(2) The time interval t' measured for an object (a subatomic particle for example) by the roadside observer is always greater than t as measured by an observer moving with the object. The only unique time interval is that measured by an observer moving with an object since all other time intervals depend on the speed of the object relative to the

observer. Thus the time t is the shortest possible and is referred to as the proper time - it is often denoted by τ.

(3) We discuss just one piece of experimental evidence for the relativity of time (we do not cite exact numerical values for pedagogical reasons). Muons are heavy versions of electrons and in earth-bound labs they decay to electrons with a half-life of about 1 μs. Muons are observed to rain down on the surface of the earth and are known to be generated in the upper atmosphere from cosmic rays. From the known height of their generation and their measured speed (close to that of light) the time for them to reach the surface of the Earth is calculated to be much longer than 1μs, let us say 10 μs (not the true value). The observed muon flux on the ground should therefore be smaller than that generated in the upper atmosphere but it is not. How can this be? The answer is the relativity of time that we have just derived. Although 10 μs has elapsed by our Earth-bound clocks the muon's time is less than 1 μs and as far as it is concerned it has not had time to decay.

Relativity of Distance

Both observers agree about the values of c and $|v|$. Thus $x/t = x'/t'$ and the relation between t and t' then implies $x' = x/\left(1-v^2/c^2\right)^{1/2}$. In this case the "proper" distance is $x' \geq x$. Lengths perpendicular to v are not affected because the perpendicular components of v are zero.