SCHWARZSCHILD RADIUS

This can be obtained by equating the Newtonian escape speed v_N to the speed of light c. To obtain v_N set the initial kinetic energy $KE = \frac{1}{2} m v_N^2$ of a mass m equal to the change in potential energy $PE = GMm/R_S$ in going from a distance R_S (the Schwarzschild radius) to infinity away from a mass M:

$$\tfrac{1}{2} m v_N^2 = \tfrac{1}{2} mc^2 = GMm/R_S \Rightarrow R_S = 2GM / c^2 . \qquad (1)$$

The term $\frac{1}{2} mc^2$ does not look relativistic and it is not, but mc^2 cannot be used because the change in potential energy $= GMm/R_S$ on the right hand side of eq. (1) is not relativistic. Surprisingly eq. (1) is the same as the general relativity (GR) result.

The Schwarzschild radius for a Planck mass $m_P = \left(\hbar c / G\right)^{1/2}$ is $R_S = \left(2Gm_P / c^2\right)$ $= 2\left(\hbar G / c^3\right)^{1/2}$, twice the Planck length.