PLANCK UNITS

These replace the SI fundamental units kg, m and s with ones that make the speed of light c, Planck's constant $\hbar = h/2\pi$, and the gravitational constant G all equal to unity. Sometimes h rather than $\hbar$ is used (I prefer $\hbar$ because ω is more fundamental than f in the relation $\hbar\omega = hf$). The derivation is simply an application of dimensional analysis. Denoting the Planck units as M_P, L_P, T_P yields

$$L_P T_P^{-1} = c; \tag{1}$$
$$M_P L_P^2 T_P^{-1} = \hbar; \tag{2}$$
$$M_P^{-1} L_P^3 T_P^{-2} = G; \tag{3}$$

The Planck mass can also be expressed as the Planck energy $E_P = M_P c^2$ and Planck temperature $\theta_P = E_P / k_B$.

Solving eqs. (1) - (3) for M_P, L_P, T_P yields

$$L_P^2 = \hbar G / c^3 = \left(1.06\times10^{-34}\right)\left(6.67\times10^{-11}\right)/\left(3\times10^{8}\right)^3 = 2.62\times10^{-70}\ \text{m}^2 \Rightarrow$$
$$L_P = 1.6\times10^{-35}\ \text{m} \tag{4}$$

$$M_P^2 = \hbar c / G = \left(1.06\times10^{-34}\right)\left(3.0\times10^{8}\right)/\left(6.67\times10^{-11}\right) = 4.77\times10^{-16}\ \text{kg}^2 \Rightarrow$$
$$M_P = 2.18\times10^{-8}\ \text{kg} \tag{5}$$

$$E_P = M_P c^2 = \left(2.18\times10^{-8}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)^2$$
$$E_P = 1.96\times10^{9}\ \text{J} = 1.23\times10^{28}\ \text{eV} \tag{6}$$

$$\theta_P = M_P / k_B = 1.96\times10^{9} / 1.38\times10^{-23} \Rightarrow$$
$$\theta_P = 1.42\times10^{32}\ \text{K} \tag{7}$$

$$T_P = L_P / c = \hbar G / c^{2.5} \Rightarrow$$
$$T_P = 2.5\times10^{-46}\ \text{s} \tag{8}$$