

On the history of the so-called Lense-Thirring effect

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Abstract

Some historical documents, especially the Einstein-Besso manuscript from 1913, an extensive notebook by Hans Thirring from 1917, and a correspondence between Thirring and Albert Einstein in the year 1917 reveal that most of the merit of the so-called Lense-Thirring effect of general relativity belongs to Einstein. Besides this “central story” of the effect, we comment shortly on some type of prehistory, with contributions by Ernst Mach, Benedikt and Immanuel Friedlaender, and August Föppl, and we follow the later history of the problem of a correct centrifugal force inside a rotating mass shell which was resolved only relatively recently. We also shortly comment on recent possibilities to confirm the so-called Lense-Thirring effect, and the related Schiff effect, experimentally.

Key words: Lense-Thirring effect, Dragging, Coriolis force, Centrifugal force, Mach’s principle

1 Introduction

For Isaac Newton the experiment with the rotating bucket was the decisive reason to introduce the concept of an absolute space. In contrast, Ernst Mach argued that also this experiment may fit under the postulate of relativity of rotation if one assumes appropriate influences of (rotating) cosmic masses on local systems. At the end of the 19th century, the brothers Benedikt and Immanuel Friedlaender considered in more detail such non-Newtonian “gravitational forces”, and they, and August Föppl even performed (unsuccessful) experiments to detect such forces.

A more systematic analysis of such additional forces began when Albert Einstein tried to generalize the Newtonian theory of gravity such that it obeys (at least locally) the principles of special relativity. On this way he first considered a scalar, relativistic gravity theory, and found therein the phenomenon of linear dragging (of test masses and inertial systems) inside a linearly accelerated mass shell. Soon afterwards he developed (with Marcel Grossmann) the tensorial Entwurf-theory, and derived now (with Michele Besso) a Coriolis type force within a rotating, spherical mass shell (which was one half of the value in final general relativity), and a motion of the nodes of planets due to the sun's rotation (which was one fourth of the value in general relativity).

Hans Thirring started in the year 1917, on the basis of general relativity, an extensive notebook "Wirkung rotierender Massen" (effect of rotating masses). The first third of this notebook considers mainly "centrifugal effects" of second order in the angular velocity ω , and is of little lasting value. Only after a letter by Einstein (dated August 2, 1917) he considers also Coriolis effects (of first order in ω), and calculates such effects near the center of a rotating mass shell, and in the far field of a rotating spherical body. Astronomical applications of these results were performed by Josef Lense. In today's literature these results run under the somewhat misleading name "Lense-Thirring effects", whereas the main merit for, and insight into this new "gravitational force" belongs to Einstein.

Thirring's notebook, and his well known publication (Thirring, 1918a) direct their main attention to a so-called centrifugal force. However, this force (as induced by rotating bodies in a laboratory, or by the rotating earth) is on one hand far below any measurability even with present technology, on the other hand it has, besides the structurally correct components, also an incorrect axial component. Also later corrections of Thirring's work by (Cornel Lanczos, 1923) and (L.Bass, & Felix Pirani, 1955) could not cure this defect. A solution of this "centrifugal force problem" in a rotating mass shell (of mass M) requires an aspherical deformation of the shell, a flat space-time in its interior, and a treatment in orders M^2 and higher. These requirements were fulfilled not earlier than in 1985 by (Herbert Pfister, & Karlheinz Braun, 1985). Herewith, the postulate of relativity of rotation was realized—within the model class of rotating mass shells—as completely as one can wish.

In recent years the so-called Lense-Thirring effect (of first order in ω) has received new interest and importance because it becomes now—more than 85 years after the theoretical predictions of Einstein, Thirring, and Lense— possible to directly measure this tiny effect. (Indirect confirmations are contained in some earlier precision tests of general relativity like Lunar Laser Ranging, and the analysis of double pulsar systems.) On one hand it has been possible to follow the orbits of the geophysical LAGEOS satellites so precisely that the motion of their nodes due to the earth's rotation showed agreement with

the predictions of general relativity within 10 % (Ciufolini, & Pavlis, 2004). On the other hand, already in the years 1959-1960 it was discovered by G. E. Pugh (1959) and Leonard Schiff (1960a,b) that the gravitomagnetic dragging phenomenon of general relativity leads to another effect—sometimes called the Schiff effect—which might be suited for experimental confirmation: The rotation axis of a gyroscope, orbiting (inside a satellite) the earth in a height of e.g. 650 km, suffers, besides other, more dominant effects, a precession of 42 milliarcseconds per year, due to the earth’s rotation. (For more details about this effect see e.g. Ignazio Ciufolini, & John Wheeler (1995), Chap.6.) At the Hansen Laboratory of Stanford University a corresponding satellite mission, Gravity Probe B, was prepared since more than 35 years, pushing the technology to its extreme in many places (Francis Everitt et al., 2001). On April 20, 2004 the satellite was successfully launched. The period of data taking is now finished, and hopefully in 2006 the results (of predicted accuracy of 1 % or better) will be communicated.

2 The prehistory (Mach, Friedlaender, Föppl)

Here we can be relatively short because in the book (Julian Barbour, & Pfister, 1995) this history is extensively treated, with longer (translated) quotations from Mach, Friedlaender, and Föppl. The idea that rotating bodies may exert on test particles not only the static (Newtonian) gravitational force but an additional “dragging force” deflecting the test particles in the direction of the rotation, was presumably first formulated by Mach (1872): “Obviously it does not matter whether we think of the earth rotating around its axis, or we imagine a static earth, and the celestial bodies rotating around it.” (This quotation can be seen as a type of definition for the “postulate of relativity of rotation”, frequently appearing in this paper.) As is well known, Mach elaborated more in detail on these questions in his famous book on mechanics (Mach, 1883): “The principles of mechanics can, presumably, be so conceived that even for relative rotations centrifugal forces arise. Newton’s experiment with the rotating vessel of water simply informs us that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick”. And although Mach did not provide a concrete extension of Newton’s laws of inertia and gravitation, e.g. by adding velocity-dependent forces, and although he did not perform any “dragging experiments”, Mach’s mechanics was a decisive stimulus for other physicists (like Friedlaender, and Föppl; see below) to do such things. Conversely, Mach reacted, in later editions of his mechanics

(e.g. in the third edition of 1897, and in the sixth edition of 1908) quite positively to these attempts.

In 1896 the brothers Friedlaender published a very interesting booklet (Friedlaender, 1896). On one hand they formulated, in extension of Mach's work, interesting, in some cases even prophetic theoretical ideas: "It seems to me that the correct form of the law of inertia will only then have been found when *relative inertia* as an effect of masses on each other and *gravitation*, which is also an effect of masses on each other, have been derived on the basis of a *unified law*". At the end of the booklet, B. Friedlaender even vaguely anticipates Einstein's incorporation of inertia and gravity into the properties of space and time: "It is also readily seen that in accordance with our conception the motions of the bodies of the solar system can be regarded as pure inertial motions, whereas in accordance with the usual conception the inertial motion, or rather its gravitationally continually modified tendency, strives to produce a rectilinear tangential motion." The concrete—but of course unsuccessful—experiment of I. Friedlaender searched for a possible influence of a rapidly rotating, heavy fly-wheel on a torsion balance mounted above the fly-wheel, in line with its axis.

A quite different, and, in principle, more promising experiment was performed by Föppl (1904). Here the rotating source was the whole earth, and the test system was a gyroscope, consisting of two heavy fly-wheels, rotating with angular velocity up to 2300 rpm. It was tested whether the rotating earth induced a Coriolis-type "dragging force" (of first order in its angular velocity ω , in contrast to the ω^2 -type effects due to a centrifugal force) on the gyroscope axis, and it was found that such an effect was less than 2 % of ω .

3 The central story (Einstein, Thirring, Lense)

As is well known, Einstein started his search for a relativistic gravitation theory in (Einstein, 1907), where he introduced the equivalence principle, and derived therefrom a gravitational redshift and a light deflection. In 1912, Einstein formulated a scalar, relativistic gravitation theory (Einstein, 1912a,b), and showed that such a theory necessarily has to be nonlinear, in order to obey the equivalence principle. Within this theory, Einstein performed the first concrete calculation of a Machian dragging effect (Einstein, 1912c)¹: First he introduced the model of an infinitely thin, spherical mass shell (mass M , radius R), a model which is very useful until today in final general relativ-

¹ The reason why this article is "hidden" in a very unusual journal is that Einstein was on very friendly terms with H. Zangger, a professor for forensic medicine in Zürich, and the quoted volume was a birthday present to Prof. Zangger.

ity, because (a) it represents the optimal substitute for the Newtonian mass point which is forbidden in general relativity due to the collapse phenomenon, (b) it allows to study mass effects by solving only the vacuum Einstein field equations (in the interior and exterior of the mass shell). Einstein then considered a test mass m at the center of this shell and derived (within his scalar theory) that the presence of the mass shell M induces an increase of m by a factor $(1 + MG/Rc^2)$. On the basis of this result he calculated that if an external force exerts a linear acceleration Γ on the mass shell, the test mass m is dragged along with the acceleration $\gamma = (3MG/2Rc^2)\Gamma$. Although these results surely encouraged Einstein in his Machian point of view on the way to a consistent relativistic theory of gravity, it has to be said that the above results are obsolete from today's knowledge: It should have been doubtful already in 1912 how a mass increase due to a gravitational field should be experimentally confirmed because such a field acts universally on all physical systems and measuring instruments. Indeed, in general relativity it was, after numerous controversial claims, shown by Carl Brans (1962) that such a mass increase is only an untestable coordinate effect. Furthermore, the title of (Einstein, 1912c) is somewhat misleading because a scalar theory can hardly produce an effect analogous to the (vectorial!) electrodynamic induction.

Very soon after the paper (Einstein, 1912c), partly still in Prague but mostly after his move to Zürich in August 1912, Einstein reached decisive new findings for a relativistic gravity theory (compare (Howard, & Stachel, 1989) for a more extensive discussion of these issues.): It should be based on a non-Euclidean (pseudo-Riemannian) geometry with metric tensor $g_{\mu\nu}$, it should be a tensorial theory with the whole energy-momentum tensor $T_{\mu\nu}$ as source of the gravitational field, and it should, if possible, be covariant with respect to general coordinate transformations. The Zürich notebook of this time (see John Norton (1984)) reveals that Einstein and the mathematician Marcel Grossmann were even considering the Ricci tensor for the "left hand side" of the field equations, and were within a hair's breadth of finding the final Einstein equations of general relativity of November 1915. But the erroneous conclusion that such a theory would not lead to the correct Newtonian limit, urged them to discard general covariance, and to propose in the so-called Entwurf-theory (Einstein, & Grossmann, 1913) for the left hand side of the field equations a "tensor" which is covariant only with respect to a reduced class of coordinate transformations.

Whereas the Entwurf-paper contains no direct applications of the new gravitation theory, such applications were performed, immediately after finishing this paper, by Einstein with his friend Michele Besso in June 1913, in the 53 pages of the so-called Einstein-Besso manuscript. (See Klein et al. (1995), pp. 344-473, where this important manuscript is reprinted, together with extended comments.) The main objective of this manuscript was the perihelion advance of Mercury which at that time was the only observation being

in conflict with Newton's gravity theory. However, within a weak-field approximation of the Entwurf-theory Einstein and Besso got for Mercury's perihelion advance a value of 18 arcseconds per century instead of the experimental value 43 arcseconds per century (more precisely, the value was $5/12$ of the value in final general relativity). This will be the reason why Einstein never published these calculations, and it surely was one of the decisive reasons for later discarding the Entwurf-theory. (It may be remarked that Johannes Droste (1914) calculated, independently of Einstein and Besso, the same "wrong" result for the perihelion shift in the Entwurf-theory.) But besides this incorrect result for the perihelion shift, the Einstein-Besso manuscript contains some other interesting and future-pointing results: On pp. 36-37 they derive a Coriolis force inside a spherical, rotating mass shell (mass M , radius R), and calculate the resulting "dragging" of test particles: For the ratio f between the induced angular velocity of the test particles and the angular velocity of the mass shell they get $f = 4MG/3Rc^2$, half the value which Thirring derived in 1918 in final general relativity (Thirring, 1918a). This is the only part of the manuscript entering Einstein's great talk in September 1913 at the Naturforscherversammlung in Vienna (Einstein, 1913a), where he also remarks that "unfortunately the expected effect is so small that we cannot hope to verify it in terrestrial experiments or in astronomy". On p. 38 of the manuscript, Einstein and Besso derive the dragging of test particles inside a linearly accelerated mass shell: $\gamma = (2MG/Rc^2)\Gamma$, a factor $4/3$ bigger than in the scalar theory of 1912 (Einstein, 1912c), and now derived without the dubious detour of a mass increase due to a gravitational field. On pp. 45-49 of the manuscript, Einstein and Besso calculate the motion of the nodes of planets in the field of the rotating sun. If one compares their result ([eq.331] on [p.49]) with the later calculation of Lense and Thirring (1918) in final general relativity (eq.(17) on p.161), and adjusts the different notations, it is seen that the effect in the Entwurf-theory is only $1/4$ of the effect in general relativity. (When Einstein and Besso calculate the effect for the planets Mercury and Venus, they get, however, much too large values because they insert a wrong value for the solar mass.)

As far as we know, in the years 1915-1916 neither Einstein nor anybody else calculated any gravitomagnetic or dragging effect in the Entwurf-theory or in general relativity. In 1917 Thirring started to calculate such effects within general relativity, which, at least partly, and up to the first order in the angular velocity ω , Einstein (and Besso) had already calculated with very similar methods and results in 1912-1913 in the scalar and/or the Entwurf-theory. The papers (Thirring, 1918a) and (Lense, & Thirring, 1918) do not clearly reveal how much knowledge Thirring had of the earlier results of Einstein. But at least Einstein's speech at the 1913 Vienna congress (Einstein, 1913a) must have been known to Thirring because he gave a talk (on specific heat of crystals) at the same congress, and Einstein's speech was the main event at this congress. Furthermore, Thirring published in this time frequently in the

