DERIVATION OF $\frac{\varepsilon_0}{\varepsilon_\infty} = \frac{\left\langle \tau_D{}^2 \right\rangle}{\left\langle \tau_D \right\rangle^2}$

$$\varepsilon' = \frac{M'}{M'^2 + M''^2}$$

$$\lim_{\omega_D \tau \to 0} M' = M_\infty \int g\left(\tau_D\right) \frac{\omega^2 \tau_D{}^2}{1 + \omega_D{}^2 \tau^2} = M_\infty \omega^2 \left\langle \tau_D{}^2 \right\rangle$$

$$\lim_{\omega_D \tau \to 0} M'' = M_\infty \int g\left(\tau_D\right) \frac{\omega \tau_D}{1 + \omega_D{}^2 \tau^2} = M_\infty \omega \left\langle \tau_D \right\rangle$$

Thus

$$\lim_{\omega \tau_D \to 0} \varepsilon' = \varepsilon_0 = \frac{M_\infty \omega^2 \left\langle \tau_D{}^2 \right\rangle}{M_\infty{}^2 \left[\omega^4 \left\langle \tau_D \right\rangle^2 + \omega^2 \left\langle \tau_D \right\rangle^2 \right]}$$

$$\rightarrow \frac{1}{M_\infty} \frac{\omega^2 \left\langle \tau_D{}^2 \right\rangle}{\omega^2 \left\langle \tau_D \right\rangle^2} = \varepsilon_\infty \frac{\left\langle \tau_D{}^2 \right\rangle}{\left\langle \tau_D \right\rangle^2}$$

QED