



Recorded Data

Measurement of Earth's Magnetic Field at Rochester, NY

The axis of a square 500 turn coil, passing through the mid points of opposite sides, was horizontal and oriented east-west. Thus the measured $|B|$ is the component pointing north.

Each side was 60 ± 0.2 cm long, as measured half way through the thickness of the windings (about 3 cm). The coil was rotated manually at as constant a rate as possible (a mechanical rotator would require an electric motor whose time dependent magnetic field would probably interfere with the earth's field). The output $V(t)$ is shown in the figure above.

The average and standard deviation of the 11 maximum voltages between 1.5 and 17 seconds are $(+0.0530 \pm 0.0033)$ V and those of the 11 minimum voltages are

(-0.0510 ± 0.0051) V. The average and standard deviation of these extrema are

$(+0.002 \pm 0.006)$ V where $0.006 = \left[(0.0033)^2 + (0.0051)^2 \right]^{1/2}$, so that the average is zero within

the uncertainty. The average and standard deviation of all 22 extreme voltage magnitudes are (0.0520 ± 0.0043) V. In summary:

Amplitude $V_{\max} = 0.052 \pm 0.004$ V,

Area $A = (0.360 \pm 0.002)$ m²,

Period $T = 1.16$ s ; angular frequency $\omega = 2\pi/T = 5.42$ rad/sec (negligible uncertainty),

Number of turns $N = 500$ turns (negligible uncertainty).

The computed magnetic field is

$$B_{\text{average}} = V_{\text{max}} / (N \cdot A \cdot \omega) = 0.052 / [(500)(0.36)(5.42)] = 5.33 \times 10^{-5} \text{ Tesla (T)}$$

and the fractional uncertainty is

$$\begin{aligned} \delta B/B &\approx \left[(\delta V/V)^2 + (\delta A/A)^2 \right]^{1/2} = \left[(0.004/0.052)^2 + (0.002/0.36)^2 \right]^{1/2} \\ &\approx (0.004/0.052) = (0.0769) \\ &\Rightarrow \delta B = (0.0769)(5.33 \times 10^{-5} \text{ T}) = 0.41 \times 10^{-6} \text{ T.} \end{aligned}$$

Thus $|B| = (5.33 \pm 0.41) \times 10^{-5} \text{ T}$

or

$$|B| = (5.3 \pm 0.4) \times 10^{-5} \text{ T.}$$

The NOAA website states that at Rochester NY $|B| = 5.378 \times 10^{-5} \text{ T}$ (it is actually given to 6 significant figures) and is directed $69^\circ 20'$ below horizontal and $11^\circ 37'$ west of north. Thus the north component is $(5.378 \times 10^{-5}) \cos(11.5^\circ) = 5.27 \times 10^{-5} \text{ T}$. The difference between the NOAA and measured field is therefore about 1%, fortuitously close compared with the estimated 8% uncertainty in the measured value. The predicted east-west component is $(5.378 \times 10^{-5}) \sin(11.5^\circ) = 1.07 \times 10^{-5} \text{ T}$ that corresponds to 0.010 V for the coil considered here.