

# Loop Quantum Gravity

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## Abstract

The problem of describing the quantum behavior of gravity, and thus understanding *quantum spacetime*, is still open. Loop quantum gravity is a well-developed approach to this problem. It is a mathematically well-defined background-independent quantization of general relativity, with its conventional matter couplings. Today research in loop quantum gravity forms a vast area, ranging from mathematical foundations to physical applications. Among the most significant results obtained so far are: (i) The computation of the spectra of geometrical quantities such as area and volume, which yield tentative quantitative predictions for Planck-scale physics. (ii) A physical picture of the microstructure of quantum spacetime, characterized by Planck-scale discreteness. Discreteness emerges as a standard quantum effect from the discrete spectra, and provides a mathematical realization of Wheeler’s “spacetime foam” intuition. (iii) Control of spacetime singularities, such as those in the interior of black holes and the cosmological one. This, in particular, has opened up the possibility of a theoretical investigation into the very early universe and the spacetime regions beyond the Big Bang. (iv) A derivation of the Bekenstein–Hawking black-hole entropy. (v) Low-energy calculations, yielding  $n$ -point functions well defined in a background-independent context. The theory is at the roots of, or strictly related to, a number of formalisms that have been developed for describing background-independent quantum field theory, such as spin foams, group field theory, causal spin networks, and others. I give here a general overview of ideas, techniques, results and open problems of this candidate theory of quantum gravity, and a guide to the relevant literature.

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# 1 Introduction

The loop approach to quantum gravity is more than twenty years old.<sup>1</sup> Today, it forms a wide research area around a well-defined tentative theory of quantum spacetime. The approach provides a candidate theory of quantum gravity, a physical picture of Planck-scale quantum geometry, calculation techniques, definite quantitative predictions, and a tool for discussing classical problems such as black-hole thermodynamics and the physics of the Big Bang.

We still do not know if the theory is *physically* correct. Direct or indirect experimental support is lacking. This is the case, unfortunately, for *all* present approaches to quantum gravity. The reason, of course, is the minuteness of the scale at which (presumably) quantum properties of spacetime manifest. Waiting for experiments, a theory must be evaluated and compared with alternatives only in terms of its consistency with what we do know about Nature, internal coherence, and its capacity to produce unambiguous novel predictions. But sound scientific standards demand that no definitive conclusion be drawn.

Although fairly well developed, loop quantum gravity (or “loop gravity”) is not yet a complete theory, nor has its full consistency with classical general relativity been rigorously established yet. The sector of the theory, which has not yet solidified, is the dynamics, which exists in several variants that are presently under investigation. The strength of the theory is its compelling capacity to describe quantum spacetime in a background-independent and nonperturbative fashion, and, especially, its genuine attempt to synthesize the conceptual novelties introduced by quantum mechanics with the ones introduced by general relativity: loop quantum gravity offers a possible conceptual framework in which general relativity and quantum field theory make sense together and consistently.

The other large research program for a quantum theory of gravity besides loop gravity, is string theory, which is a tentative theory as well. String theory is more ambitious than loop gravity, since it also aims at unifying all known fundamental physics into a single theory. In Section 2.3, I compare strengths and weaknesses of these two competing approaches to quantum gravity.

This “living review” is intended to be a tool for orienting the reader in the field of loop gravity. Here is the plan for the review:

- Section 2, “**Quantum Gravity: Where Are We?**”, is an introduction to the problem, the reason for its relevance, and the present state of our knowledge.
- Section 3, “**The Development of Loop Gravity**”, is a short overview of the historical development of the theory.
- Section 4, “**Resources**”, contains pointers to introductory literature, institutions at which loop gravity is studied, web pages, and other information that may be of use to students and researchers.
- Section 5, “**Ideas and Physical Inputs**”, discusses the physical and mathematical ideas on which loop quantum gravity is based.
- The actual theory is introduced in Section 6, “**Formalism**”, at a simple, technical level.
- Section 7, “**Physical Results**”, is a list of the main physical results that have been derived from the theory.
- Section 8, “**Open Problems and Current Lines of Investigation**”, illustrates the main open problems and some currently active lines of research.

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<sup>1</sup>The first talk on “a loop space representation of quantum general relativity” was given at a conference in India in 1987 [264].

- Section 9, “**Short Summary and Conclusion**”, summarizes very briefly the state and the results of the theory.

At the cost of several repetitions, the structure of this review is modular: to a large extent sections are independent of one another, have different style, and can be combined according to the interest of the reader. A reader interested only in a very brief overview of the theory and its results can find this in Section 9. Graduate students and nonspecialists may get a general idea of what goes on in this field and its main ideas from Sections 2 and 7. If interested only in the technical aspects of the theory and its physical results, one can read Sections 6 and 7 alone. Scientists working in this field can use Sections 6 and 7 as a reference, and I hope they will find Sections 2, 5 and 8 stimulating. I will not enter into technical details. I will point to the literature where these details are discussed. I have tried to be as complete as possible in indicating all relevant aspects and potential difficulties of the issues discussed.

The literature in this field is vast, and I am sure that there are works whose existence or relevance I have failed to recognize. I sincerely apologize to the authors whose contributions I have neglected or under-emphasized, and I strongly urge them to contact me to help me make this review more complete. The “living reviews” are constantly updated, and I can correct errors and omissions.

## 2 Quantum Gravity: Where Are We?

This is a nontechnical section in which I illustrate the problem of quantum gravity in general, its origin, its importance, and the present state of our knowledge in this regard.

The problem of describing the quantum regime of the gravitational field is still open. There are tentative theories and competing research directions. For an overview, see [258, 155]. The book [218] presents a large and interesting spectrum of viewpoints and opinions. The two largest research programs are string theory and loop gravity. Examples of other directions explored are noncommutative geometry [89], causal dynamical triangulations [13], causal sets [279], twistor theory [225], doubly-special relativity [166], and Euclidean quantum gravity [141, 144]. Research directions are variously related; in particular, formalisms such as spin foams (Section 6.7), group field theory (Section 6.8), or uniform discretizations (Section 6.10) are variously viewed as strictly related to loop gravity or independent research directions.

String theory and loop gravity differ not only because they explore distinct physical hypotheses, but also because they are expressions of two separate communities of scientists, which have sharply distinct prejudices, and who view the problem of quantum gravity in surprisingly different manners.<sup>2</sup>

### 2.1 What is the problem? The view of a high-energy physicist

High-energy physics has obtained spectacular successes during the last century, culminating with the laborious establishment of quantum field theory as the general form of dynamics and with the extraordinary and unexpected success of the  $SU(3) \times SU(2) \times U(1)$  standard model. This success is now several decades old. Thanks to it, physics is in a position in which it has been rarely: there are virtually no experimental results that clearly challenge, or clearly escape, the present fundamental theory of the world. The theory we have encompasses virtually everything – except gravitational phenomena.<sup>3</sup> From the point of view of a particle physicist, gravity is then simply the last and the weakest of the interactions. The problem of quantum gravity is perceived as a last step in the path towards unification. It is then natural to try to understand the quantum properties of gravity using the strategy that has been so successful for the rest of microphysics, or variants of this strategy.

The search for a conventional quantum field theory capable of embracing gravity has spanned several decades and, through an adventurous sequence of twists, moments of excitement and bitter disappointments, has led to string theory. The foundations of string theory are not yet well understood; and it is not entirely clear how the current versions of the theory, which are supersymmetric and formulated in 10 or 11 dimensions, can be concretely used for deriving comprehensive univocal predictions about *our* world. But string theory may claim remarkable theoretical successes and is today the most widely studied candidate theory of quantum gravity.

In string theory, gravity is just one of the excitations of a string or other extended object, living on some metric space. The existence of such background spaces, in which a theory is defined, is the key technical tool for the formulation and the interpretation of the theory, at least in the case of the perturbative definition of the theory. In tentative nonperturbative definitions, such as aiming to define the physical theory indirectly via a boundary quantum field theory, the theory relies only on the background “at infinity”, needed for the definition of the boundary quantum field theory.

In all cases, for a physicist with a high-energy background, the central problem of quantum gravity is reduced to an aspect of the problem of understanding the still mysterious nonperturbative theory that has the various perturbative theories as its perturbation expansion.

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<sup>2</sup>The relative size of the two communities can be estimated from the fact that there were 452 registered participants at the STRINGS 07 conference in Madrid, EU, and 156 at the LOOPS 07 conference in Morelia, Mexico.

<sup>3</sup>This situation might change with the data from the LHC, expected soon.

## 2.2 What is the problem? The view of a relativist

For a relativist, on the other hand, the idea of a fundamental description of gravity in terms of physical excitations over a background space sounds physically wrong. The key lesson learned from general relativity is that there is no background metric space *over* which physics happens (except, of course, in approximations). The world is more complicated, or perhaps simpler, than that. For a relativist, in fact, general relativity is much more than the field theory of one particular force. Rather, it is the discovery that certain classical notions about space and time are inadequate at the fundamental level: they require modifications, which are possibly as basic as those introduced by quantum mechanics. One of these inadequate notions is precisely the notion of a background space (flat or curved), in which physics happens. This profound conceptual shift, which has led to the understanding of relativistic gravity, the discovery of black holes, relativistic astrophysics and modern cosmology, is now considered by relativists to be acquired knowledge about the world.

From Newton to the beginning of the last century, physics has had a solid foundation in a small number of key notions such as space, time, causality and matter. In spite of substantial evolution, these notions have remained rather stable and self-consistent. In the first quarter of the last century, quantum theory and general relativity have deeply modified this foundation. The two theories have obtained solid success and vast experimental corroboration, and can now be considered well established. Each of the two theories modifies the conceptual foundation of classical physics in a (more or less) internally-consistent manner, but we do not have a novel conceptual foundation capable of supporting *both* theories. This is why we do not yet have a theory capable of predicting what happens in the physical regime in which both theories are relevant, the regime of Planck-scale phenomena,  $10^{-33}$  cm.

General relativity has taught us not only that space and time share the property of being dynamical with the rest of the physical entities, but also – more crucially – that spacetime location is relational (see Section 5.3). Quantum mechanics has taught us that any dynamical entity is subject to Heisenberg’s uncertainty at small scale. Therefore, we need a *relational notion of a quantum spacetime* in order to understand Planck-scale physics.

Thus, for a relativist, the problem of quantum gravity is the problem of bringing a vast conceptual revolution, begun with quantum mechanics and general relativity, to a conclusion and to a new synthesis.<sup>4</sup> In this synthesis the notions of space and time need to be deeply reshaped in order to take into account what we have learned with both our present “fundamental” theories.

The difference between the formulation of the problem of quantum gravity given by a high-energy physicist and a relativist derives therefore from a different evaluation of general relativity. For the first, it is just one additional field theory with a funny gauge invariance; for the second, it is a complete modification in the way we think about space and time.

This issue is often confused with the issue of whether the Einstein equations are low-energy equations that need to be corrected at high energy. But the two issues are not related: many relativists expect that the Einstein equations may very well require corrections at high energy. However, they do not expect that the corrected theory will mean a return to the old pre-general-relativistic notions of space and time.

Unlike string theory, loop quantum gravity has a direct fundamental formulation, in which the degrees of freedom are clear, and which does not rely on a background spacetime. Loop quantum gravity is thus a genuine attempt to grasp what quantum spacetime is at the fundamental level. Accordingly, the notion of spacetime that emerges from the theory is profoundly different from the one on which conventional quantum field theory or string theory is based.

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<sup>4</sup>For a discussion of this idea, see [256], [276] and [261].

